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1996 J. Phys.: Condens. Matter 8 2095

(http://iopscience.iop.org/0953-8984/8/12/022)

J. Phys.: Condens. Matter **8** (1996) 2095–2099. Printed in the UK

COMMENT

Comparison between two schemes for determining stable ground states in quantum dots

T Kawamura and J K Jain

Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794-3800

Received 18 July 1995

Abstract. We make a comparative study of two schemes that predict the ground states of a quantum dot with parabolic confinement in a strong magnetic field. One method is based on the composite fermion (CF) approach, and the other was recently proposed by Dharma-wardana. The prediction of these two schemes differ in general, and we compare the results of both methods against exact diagonalization calculations.

Dharma-wardana [1] has recently constructed a scheme for identifying the quantum numbers of the stable ground states of a quantum dot. It obtains the previously known ground state quantum numbers for five and six electrons in a parabolic quantum dot. For a more rigorous test, we carry out numerical calculations for bigger systems, and find that Dharma-wardana's prescription is not able to account for some of the stable ground states, and at the same time also predicts states which are not stable. In contrast, the composite fermion picture continues to provide a reasonable description.

We consider here the usual model in which N two-dimensional electrons move in a parabolic confining potential in the presence of a strong magnetic field. The angular momentum L is a good quantum number in this geometry, and will be used to label the states. In the limit when the confinement energy is small compared to the cyclotron energy, the eigenenergies separate into two parts, the confinement energy and the interaction energy. The confinement energy is explicitly known as a function of L. Therefore, it is sufficient to consider only the interaction energy, which, apart from a renormalized length scale, happens to be the same as the interaction energy of electrons in the absence of confinement. We will only consider the interaction energy below. The single-particle eigenstates and the Coulomb matrix elements have been discussed in detail in the past and will not be repeated here; the interested reader is referred to the literature [2–6].

We start with a summary of the results of the two schemes. Let us define

$$L_{\min} = \tfrac{1}{2}N(N-1) \tag{1}$$

which is the total angular momentum of the lowest filled Landau level (LL) state. Dharma-wardana's prescription is as follows. For any given $L > L_{\min}$, consider $L' = L - kL_{\min}$, where k, either an odd or an even integer, is chosen appropriately to make $0 < L' \leqslant L_{\min}$. Then define

$$\nu'^{-1} = L'/L_{\min} \equiv p/q. \tag{2}$$

(Dharma-wardana denotes this by ν', but we use ν'^{-1}, since it is actually the *inverse* of the filling factor.) It is claimed in [1] that a stable ground state is obtained whenever p/q

0953-8984/96/122095+05$19.50

is a rational with q odd. This picture is motivated by the idea [7] in which each electron is decomposed into fictitious particles, and a stable ground state is obtained when each species of the fictitious particles occupies a stable state. However, there are two conceptual difficulties with the prescription of [1]. (i) Allowing odd integer values for k is tantamount to assuming bosonic statistics for electrons (since each electron is decomposed into an even number of number of fictitious particles, each assumed to be a fermion). In fact, this leads to an incompressible state at $\nu = 1/2$ [1]. (ii) It would seem to us more natural to demand that p is odd, rather than q. In any case, the scheme of [1] makes precise predictions for bigger systems that are testable.

The CF scheme [8] relates the system of interacting electrons at angular momentum L to that of non-interacting composite fermions at angular momentum L^*, given by

$$L^* = L - 2L_{\min}. \tag{3}$$

In particular, the ground state energy of interacting electrons in the range $L_{\min} < L < 3L_{\min}$ resembles the ground state energy of non-interacting fermions in the range $-L_{\min} < L^* < L_{\min}$ [9–11]. The former depends on the Coulomb interaction between electrons, the evaluation of which requires a numerical diagonalization of the Hamiltonian. The latter, proportional to the kinetic energy of fermions, is determined rather straightforwardly by asking what arrangement of fermions in different LL will give the lowest kinetic energy for a given L. The LL spacing is treated as a parameter in the theory, for which the empirical formula given in [10] will be used below.

We determine the lowest energy in a given L subspace from the exact diagonalization calculations in a range of L for seven electrons in a quantum dot. The interaction energy of the state is shown in figure 1. The dimension of the largest matrix diagonalized was 3539 for $L = 56$. The actual overall ground state of the quantum dot is to be determined only after adding to the interaction energy the confinement energy, *which is a linear function of* L. Clearly, only a state with a downward cusp (DC) in figure 1 has a chance of becoming the ground state, as various parameters (confinement strength or the magnetic field) are varied. However, not all such states actually become ground states; a cusp which lies *above* (or on) the line joining any two equidistant cusps on either side will not become the ground state for any choice of parameters. The possible ground states in the L range considered in our study are seen to be at $L = 28$, 33, 39, 45, 51 and 56. These should be compared to $L = 28$, 35, 42, 49, 51 and 54 predicted by Dharma-wardana in [1].

Figure 1 also shows the composite fermion prediction, which evidently captures the major features of the exact results. We determine the possible ground states as before, and obtain $L^* = -14$, -9, -3, 3, 9 and 14, predicting ground states for the interacting electron system at $L = L^* + 42 = 28$, 33, 39, 45, 51 and 56, in impressive agreement with the exact result! (Note that the downward cusps at $L^* = -6$, -1, 1 and 6 lie *on* the line joining two equidistant cusps on either side, and hence do not become ground states.)

In fact, the CF scheme works here better than one has the right to expect, given that the above model makes the assumption of treating the composite fermions as strictly non-interacting. The interaction between the composite fermions is much weaker than that between electrons, since a good part of the interelectron interaction has been used up in the formation of composite fermions. However, neglect of the residual inter-CF interaction is only a good zeroth-order model; it gets the principal features correctly, but will miss secondary structure attributable to the residual interaction between the composite fermions. For example, it is entirely possible that a state which has no cusp in the non-interacting-CF model will develop a downward cusp when the interaction between the composite fermions is switched on. However, the secondary structure is expected to be relatively weak. This is

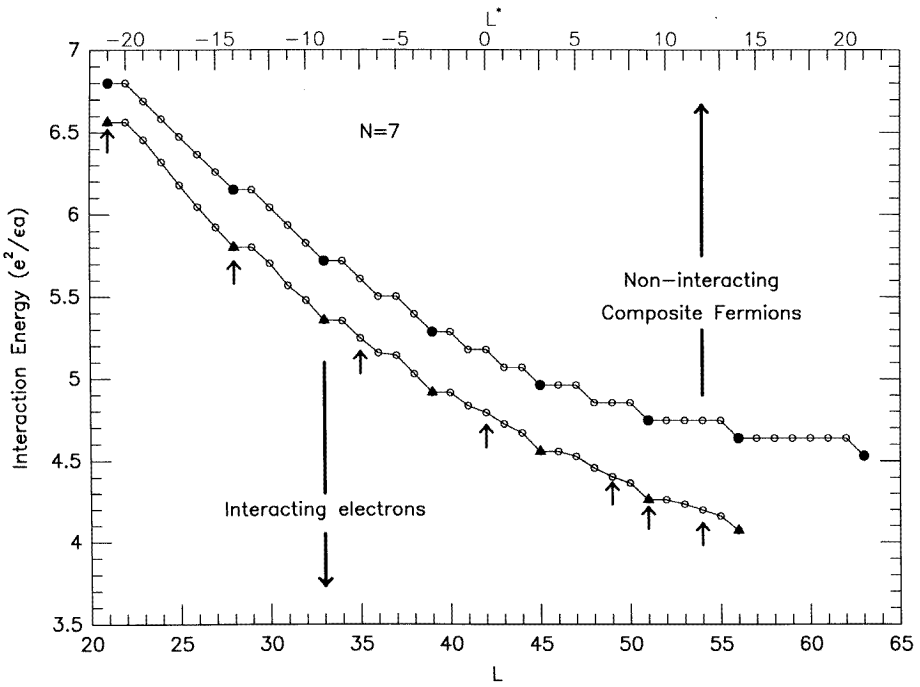

Figure 1. The lower curve shows the exact diagonalization results for the interaction energy of $N = 7$ interacting electrons as a function of the total angular momentum L (lower axis). The energy is given in units of $e^2/\epsilon a$, where ϵ is the dielectric constant of the background material and a is the renormalized magnetic length (see [6, 10, 11] for details). The upper curve shows the kinetic energy of non-interacting fermions as a function of L^* (upper axis). The effective LL spacing for the composite fermions has been determined from the expression given in [10]. The CF curve has been vertically offset for clarity. The possible ground states obtained in the exact diagonalization study are indicated by filled triangles. Filled circles show the possible ground states predicted by the CF theory; Dharma-wardana's predictions [1] are indicated by upward arrows.

indeed the case. For example, for six electrons, the numerical calculations obtain a ground state at $L = 40$, not predicted in the non-interacting-CF picture, but this ground state appears only in an extremely narrow range of parameters [5] and has a very weak cusp associated with it. Another example is provided in the ten-electron results of figure 2, where the state at $L = 61$ becomes the ground state in a (small) range of parameters, not predicted by the non-interacting-CF model.

What is the residual interaction between the composite fermions? The answer is likely to be rather complex. The CF scheme, however, does allow the computation of the weaker structure as well, albeit only with substantially greater effort. It would involve constructing the CF basis at each L, obtained by starting from the basis states of non-interacting electrons at $L^* = L - 2L_{\min}$, multiplying them by an appropriate Jastrow factor, and then projecting the resulting state on to the lowest LL. The CF theory asserts that a good description of the low energy states is provided by diagonalizing the Coulomb Hamiltonian in this basis. For most DC states, the CF wavefunction is unique and no diagonalization is necessary. The CF wavefunction has been studied in a few such cases and found to be quite accurate [12], implying that the CF scheme will predict the energy quite reliably. For other L, there are

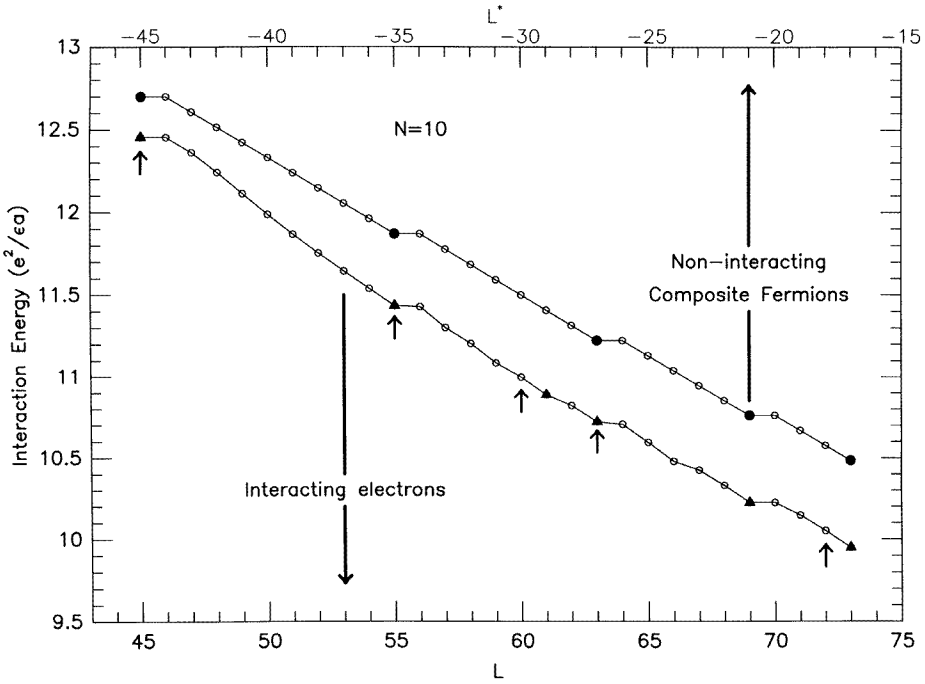

Figure 2. Same as in figure 1 for $N = 10$.

several CF states, but the CF basis is still tremendously small compared to the full lowest LL basis. While the smaller basis size helps, it is in practice rather difficult to carry out the projection on to the lowest Landau level. It should be emphasized, though, that a number of simplifying approximations have already been made in writing the initial idealized model (parabolic confinement, lack of LL mixing, and so on), a relaxation of which, required for comparison with the actual experiments, will surely alter the minor details of the solution. It is hoped that the principal structure, described well by the simple non-interacting-CF model, will survive and be observed in quantum dots.

Acknowledgments

This work was supported in part by the Office of Naval Research under Grant no. N00014-93-1-0880. We are grateful to R K Kamilla for useful discussions.

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